

层状地基中单桩扭转振动特性分析

王国才*, 丁翠红, 王 哲, 黄 晋

(浙江工业大学 建筑工程学院, 杭州 310014)

摘 要:采用积分变换和 Muki 的方法求解了层状地基中单桩的扭转振动问题。在分析过程中,首先对基本控制方程进行 Hankel 变换,建立了单层地基的初参数解答和刚度矩阵,得到层状地基的递推矩阵;然后利用递推矩阵、边界条件和桩-土变形协调条件建立了层状地基中单桩扭转振动问题的基本积分方程并进行数值求解。文末数值算例给出了退化的层状地基中刚性单桩的扭转变形,其结果与已有经典解答吻合良好。同时,并研究了两层地基中单桩的扭转动力响应,分析了桩-土参数对动力响应的影响,所得结论对工程实践和桩基扭转波检测有一定的指导意义。

关键词:层状地基;单桩;扭转振动;动力柔度系数;积分变换

中图分类号:TU435, TU473.1 **文献标识码:**A

1 引 言

随着我国桥梁工程、高层建筑等的高速发展,桩基日益得到了广泛应用。研究桩承结构物在动力荷载(如地震、风浪、机器振动等)作用下的动力响应,求得桩-土体系的动力特性是解决桩-土动力相互作用问题的主要内容,是当前土动力学中一个十分重要而又非常复杂的研究课题。

近几十年来,国内外学者围绕着桩-土动力相互作用问题展开了广泛的研究工作,提出了许多简化的计算模型及其相应的分析方法^[1-6],得出了许多重要的结论。然而,实际的地基土在其形成过程以及后来的人为因素,使得其具有较明显的成层性,所以用匀质地基模型来研究桩的响应问题有一定的局限性。层状地基模型由于能较合理地模拟地基土物理力学特性沿深度方向的不均匀性而被广泛应用于岩土工程、地震工程等领域。基于这种模型,文献[7-10]分别求得了成层土中桩纵向和水平振动半解析解。考虑到地基土的成层性,本文拟采用积分变换和 Muki 的方法研究层状地基中单桩的扭转振动问题。在分析过程中,首先对基本控

制方程进行 Hankel 变换,建立了单层地基的初参数解答和刚度矩阵,得到层状地基的递推矩阵。然后利用递推矩阵、边界条件和桩-土变形协调条件建立了层状地基中单桩扭转振动问题的基本积分方程并进行求解。文末数值算例给出了退化的层状地基中刚性单桩的扭转变形,其结果与已有经典解答吻合良好。同时,并研究了两层地基中单桩的扭转动力响应,分析了桩-土参数对动力响应的影响,所得结论对工程实践和桩基扭转波检测有一定的指导意义。

2 层状地基基本方程的求解

2.1 单层地基的传递矩阵

图 1 所示为一垂直单桩置于一 n 层地基中,桩顶承受一扭矩 $T_0 e^{i\omega t}$ (ω 为振动圆频率)的作用。在圆柱坐标系下,结合所研究问题的对称性并考虑到所有运动都是简谐运动,则土体仅有切向位移 $u_\theta e^{i\omega t}$,其余位移分量皆为零。这样,单层地基的波动方程和物理方程可写成:

$$\frac{\partial^2 \bar{u}_\theta}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}_\theta}{\partial \bar{r}} - \frac{1}{\bar{r}^2} \bar{u}_\theta + \frac{\partial^2 \bar{u}_\theta}{\partial \bar{z}^2} = -\delta^2 \bar{u}_\theta \quad (1)$$

$$\bar{\tau}_{z\theta} = \frac{\partial \bar{u}_\theta}{\partial \bar{z}}, \quad \bar{\tau}_{r\theta} = \frac{\partial \bar{u}_\theta}{\partial \bar{r}} - \frac{\bar{u}_\theta}{\bar{r}} \quad (2,3)$$

式中无量纲参数和变量的定义为 $\bar{r}=r/a, \bar{z}=z/a, \bar{u}_\theta=u_\theta/a, \delta=a\omega/\sqrt{\rho/\mu}, \bar{\tau}_{z\theta}=\tau_{z\theta}/\mu, \bar{\tau}_{r\theta}=\tau_{r\theta}/\mu$, 其中 ρ 和 μ 分别为土体的质量密度和剪切模量。

收稿日期:2007-04-18;修改稿收到日期:2008-08-20.

基金项目:浙江省自然科学基金(Y105480);浙江省教育厅(20051414)资助项目.

作者简介:王国才*(1971-),男,博士,副教授
(E-mail:wanggcjzt@hotmail.com);
丁翠红(1961-),女,硕士,副教授.

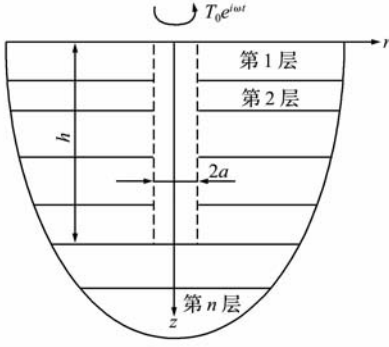


图 1 层状地基中的受扭振动单桩

Fig. 1 Single torsional pile in layered foundation

对式(1)和式(2)进行一阶 Hankel 变换并求

解,得

$$\tilde{u}_\theta^1(\bar{z}, p) = A_1 \sinh j\bar{z} + A_2 \cosh j\bar{z} \quad (4)$$

$$\tilde{\tau}_{z\theta}^1(\bar{z}, p) = A_1 j \cosh j\bar{z} + A_2 j \sinh j\bar{z} \quad (5)$$

式中,“~”表示 Hankel 变换, $j^2 = p^2 - \delta^2$, A_1 和 A_2 为与 p 有关的积分常数。需要指出的是,式(4)和(5)中既有上行波的存在,也有下行波的存在。

令式(4)和(5)两边 $\bar{z} = \bar{z}_0$, 解出 A_1 和 A_2 , 并回代到式(4)和式(5),整理后写成矩阵形式,有

$$\bar{B}(\bar{z}, p) = \phi(\bar{z}, p) \bar{B}(\bar{z}_0, p) \quad (6)$$

式中 $\bar{B}(\bar{z}, p) = [\tilde{u}_\theta^1(\bar{z}, p), \tilde{\tau}_{z\theta}^1(\bar{z}, p)]^T$, $\bar{B}(\bar{z}_0, p) = [\tilde{u}_\theta^1(\bar{z}_0, p), \tilde{\tau}_{z\theta}^1(\bar{z}_0, p)]^T$, $\phi(\bar{z}, p) = [\phi_{ij}]$ 为 2×2 阶单层地基的递推矩阵,其元素为 $\phi_{11} = \cosh j(\bar{z} - \bar{z}_0)$,

$$\phi_{12} = \frac{1}{j} \sinh j(\bar{z} - \bar{z}_0), \phi_{21} = j \sinh j(\bar{z} - \bar{z}_0),$$

$$\phi_{22} = \phi_{11}。$$

当介质为匀质半空间时,则仅有下行波,故由 $\bar{z} \rightarrow \infty$ 时的辐射条件及式(4)和式(5),可得

$$\bar{S}_B(\bar{z}, p) = -j \bar{U}_B(\bar{z}, p) \quad (7)$$

式中 $\bar{S}_B(\bar{z}, p)$ 为 $\bar{z}$ 处的应力向量, $\bar{U}_B(\bar{z}, p)$ 为 $\bar{z}$ 处的位移向量。

2.2 混合边值条件

考虑任意 n 层地基,在其中的第 m 层内部作用一单位简谐扭矩 $e^{i\omega t}$,荷载作用面半径为 a ,如图 2 所示。 μ_i 和 ρ_i 分别为第 i 层地基土的剪切模量和质量密度, h_i 为第 i 层地基土底面到地表面的距离($\bar{h}_i = h_i/a$), $\Delta h_i = h_i - h_{i-1}$ 为第 i 层地基土的厚度($\Delta \bar{h}_i = \Delta h_i/a$)。由于荷载作用平面处的应力有跳跃,将该处划分为一个分界面,并将该面之上与之下的分层分别记为 m_1 层和 m_2 层,最底面的一层地基土为半无限层。

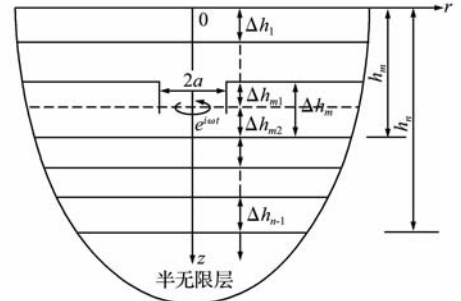


图 2 层状地基力学模型

Fig. 2 Physical model of layered foundation

将式(6)应用于前 $n-1$ 层,式(7)应用于第 n 层,则有以下的递推关系:

$$\bar{B}(\bar{h}_1^-, p) = \phi(\Delta \bar{h}_1, p) \bar{B}(0, p)$$

$$\bar{B}(\bar{h}_2^-, p) = \phi(\Delta \bar{h}_2, p) \bar{B}(\bar{h}_1^+, p)$$

$$\dots \dots$$

$$\bar{B}(\bar{h}_{m_1}^-, p) = \phi(\Delta \bar{h}_{m_1}, p) \bar{B}(\bar{h}_{m-1}^+, p)$$

$$\bar{B}(\bar{h}_m^-, p) = \phi(\Delta \bar{h}_{m_2}, p) \bar{B}(\bar{h}_{m_1}^+, p)$$

$$\dots \dots$$

$$\bar{B}(\bar{h}_n^-, p) = \phi(\Delta \bar{h}_n, p) \bar{B}(\bar{h}_{n-1}^+, p)$$

$$\bar{B}(\bar{h}_n^+, p) = \left\{ \begin{array}{l} \bar{U}_B(\bar{h}_n^+, p) \\ \bar{S}_B(\bar{h}_n^+, p) \end{array} \right\} = \left\{ \begin{array}{l} \bar{U}_B(\bar{h}_n^+, p) \\ -j \bar{U}_B(\bar{h}_n^+, p) \end{array} \right\}$$

各相邻土层间的边界接触条件如下:

$$\bar{B}(\bar{h}_j^+, p) = H_j \bar{B}(\bar{h}_j^-, p)$$

$$(j = 1, \dots, j \neq m_1) \quad (9)$$

$$\bar{B}(\bar{h}_{m_1}^+, p) = \bar{B}(\bar{h}_{m_1}^-, p) + \left[0, \frac{2}{\pi p} J_2(p) \right]^T \quad (10)$$

式中 $H_j = \text{diag}\{1, \mu_{j+1}/\mu_j\}$ 为 2×2 阶对角矩阵。

2.3 层状矩阵的递推求解

对荷载作用平面之上与之下的应力和位移,可采用不同的递推过程求解。对于荷载作用平面以上各层的未知量,其递推过程如下:

$$\bar{B}(\bar{h}_n^+, p) = \left\{ \begin{array}{l} \bar{U}_B(\bar{h}_n^+, p) \\ -j \bar{U}_B(\bar{h}_n^+, p) \end{array} \right\} =$$

$$F(\bar{h}_n, \dots, \bar{h}_1, p) \bar{B}(0, p) + G(\bar{h}_n, \dots, \bar{h}_1, p)$$

$$(11)$$

式中

$$\begin{aligned} F(\bar{h}_n, \dots, \bar{h}_1, p) &= \phi(\Delta \bar{h}_n, p) H_{n-1} \phi(\Delta \bar{h}_{n-1}, p) \cdots \times \\ &H_{m+1} \phi(\Delta \bar{h}_{m+1}, p) H_m \phi(\Delta \bar{h}_{m_2}, p) \phi(\Delta \bar{h}_{m_1}, p) \times \\ &H_{m-1} \phi(\Delta \bar{h}_{m-1}, p) \cdots H_1 \phi(\Delta \bar{h}_1, p) = \\ &\begin{bmatrix} a_1(\bar{h}_1, \dots, \bar{h}_n, p) & b_1(\bar{h}_1, \dots, \bar{h}_n, p) \\ c_1(\bar{h}_1, \dots, \bar{h}_n, p) & d_1(\bar{h}_1, \dots, \bar{h}_n, p) \end{bmatrix} \end{aligned}$$

$$G(\bar{h}_n, \cdots, \bar{h}_{m2}, p) = \left\{ \begin{aligned} &g_1(\bar{h}_n, \cdots, \bar{h}_{m2}, p) \\ &g_2(\bar{h}_n, \cdots, \bar{h}_{m2}, p) \end{aligned} \right\} = \\ \phi(\Delta \bar{h}_n, p) H_{n-1} \phi(\Delta \bar{h}_{n-1}, p) \cdots \\ H_{m+1} \phi(\Delta \bar{h}_{m+1}, p) \times \\ H_m \phi(\Delta \bar{h}_{m2}, p) \left[0, \frac{2}{\pi p} J_2(p) \right]^T$$

由于地基表面无荷载作用,则 $\tilde{\tau}_{z\theta}^1(0, p) = 0$,代入式(11),可解出

$$\tilde{u}_{z\theta}^1(0, p) = -\frac{g_1 j + g_2}{c_1 + j a_1} \tag{12}$$

故荷载作用平面以上第 i 层地基深度 z 处的位移和应力可写成:

$$\bar{B}(\bar{z}, p) = \phi(\bar{z} - \bar{h}_{i-1}, p) H_{i-1} \phi(\Delta \bar{h}_{i-1}, p) \cdots \times \\ H_1 \phi(\Delta \bar{h}_1, p) \bar{B}(0, p) \tag{13}$$

对于荷载作用平面以下第 i 层地基深度 z 处的位移和应力,采用同样类似的递推过程,得

$$\bar{B}(\bar{z}, p) = \phi(\bar{z} - \bar{h}_{i-1}, p) K_{i+1} \phi(-\Delta \bar{h}_{i+1}, p) \cdots \times \\ K_n \phi(-\Delta \bar{h}_n, p) \bar{B}(\bar{h}_n^+, p) \tag{14}$$

式中 $K_j = \text{diag}\{1, \mu_{j-1}/\mu_j\}$ 为 2×2 阶对角矩阵。

3 层状地基中单桩的扭转振动

n 层地基中受扭振动单桩如图 1 所示。桩顶作用的荷载为 $T_0 e^{i\omega t}$,桩的半径为 a ,长为 h ,剪切模量为 μ_p ,质量密度为 ρ_p ,第 i 层地基土的剪切模量为 μ_i ,质量密度为 ρ_i ,厚度为 h_i ,第 n 层地基为半无限层。

根据 Muki 的方法,图 1 所示的问题可以看成是图 3(a)与图 3(b)的叠加。其中,图 3(a)为扩展的半空间,而图 3(b)为一虚拟桩,其剪切模量 μ_v 及质量密度 ρ_v 分别为

$$\mu_v = \mu_p - \mu_i, \quad \rho_v = \rho_p - \rho_i \tag{15,16}$$

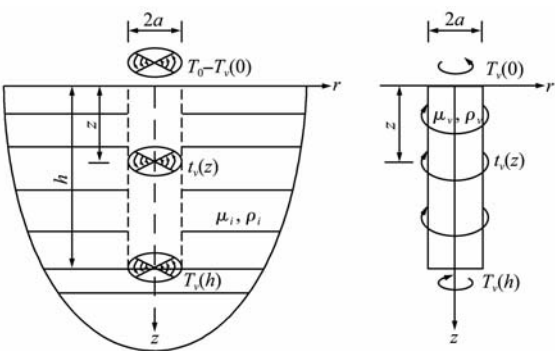
考虑到稳态振动,虚拟桩的扭转角 $\phi_v(\bar{z})$ 和沿桩身的分布剪力 $\bar{t}_v(\bar{z})$ 有如下关系:

$$\bar{\mu}_v J \frac{\partial^2 \phi_v(\bar{z})}{\partial \bar{z}^2} + \bar{t}_v(\bar{z}) + \bar{\rho}_v J \delta^2 \phi_v(\bar{z}) = 0 \\ (0 \leq \bar{z} \leq \bar{h}) \tag{17}$$

而虚拟桩的扭矩 $\bar{T}_v(\bar{z})$ 和扭转角 $\phi_v(\bar{z})$ 有如下的关系:

$$\bar{T}_v(\bar{z}) = \bar{\mu}_v J \frac{d\phi_v(\bar{z})}{d\bar{z}} \quad (0 \leq \bar{z} \leq \bar{h}) \tag{18}$$

式中 $J = \pi/2, \bar{t}_v(\bar{z}) = t_v(z)/(\mu_1 a^3), \bar{\mu}_v = \mu_v/\mu_1, \bar{\rho}_v = \rho_v/\rho_1, \bar{T}_v(\bar{z}) = T_v(z)/(\mu_1 a^3)$ 。



(a) 扩展的半空间 (b) 虚拟桩
(a) Extended half space (b) Virtual pile

图 3 层状地基单桩扭转振动计算模型
Fig. 3 Calculating model of torsional vibration of single pile embedded in layered foundation

作用在扩展半空间上的力有:扭矩 $\bar{T}_0 - \bar{T}_v(0)$ 和 $\bar{T}_v(\bar{h})$ 以及分布剪力 $\bar{t}_v(\bar{z})$ 。因此,扩展半空间任一点的位移可表示为

$$\bar{u}_\theta(\bar{r}, \bar{z}) = [\bar{T}_0 - \bar{T}_v(0)] \hat{u}_{\theta T}(\bar{r}, \bar{z}; 0) + \\ \bar{T}_v(\bar{h}) \hat{u}_{\theta T}(\bar{r}, \bar{z}; \bar{h}) + \int_0^{\bar{h}} \hat{u}_{\theta T}(\bar{r}, \bar{z}; \bar{z}') \bar{t}_v(\bar{z}') d\bar{z}' \tag{19}$$

式中 $\bar{T}_0 = T_0/(\mu_1 a^3), \hat{u}_{\theta T}(\bar{r}, \bar{z}; \bar{z}')$ 为在截面 $\Pi_{z'}$ 处受到单位简谐荷载作用而引起扩展半空间在截面 $\Pi_{\bar{z}}$ 处所对应的切向位移。

假设桩作扭转振动时,桩与土不脱离,桩土之间完全粘结在一起,有

$$\bar{u}_\theta(1, \bar{z}) = 1 \cdot \phi_v(\bar{z}) \tag{20}$$

根据式(17)~式(20),可导出控制层状地基中单桩扭转振动的第二类 Fredholm 积分方程:

$$\left(\frac{1}{\bar{\mu}_v J} + 2\hat{\gamma}_1(\bar{z}, \bar{z}) \right) \bar{\bar{T}}_v(\bar{z}) = \\ \hat{\gamma}_T(\bar{z}, 0) + \int_0^{\bar{h}} \left[\bar{\bar{T}}_v(\zeta) \frac{\partial \hat{\gamma}_T(\bar{z}, \zeta)}{\partial \zeta} d\zeta + \right. \\ \left. \frac{\bar{\rho}_v \delta^2}{\bar{\mu}_v} \int_0^{\bar{h}} \hat{\gamma}_T(\bar{z}, \zeta) \int_{\bar{\zeta}}^{\bar{h}} \bar{\bar{T}}_v(\zeta') d\zeta' d\zeta - \right. \\ \left. \Delta \int_0^{\bar{h}} \hat{\gamma}_T(\bar{z}, \zeta) d\zeta \left\{ \hat{\phi}_T(\bar{h}, 0) + \right. \right. \\ \left. \left. \int_0^{\bar{h}} \left[\bar{\bar{T}}_v(\zeta) \frac{\partial \hat{\phi}_T(\bar{h}, \zeta)}{\partial \zeta} + \right. \right. \right. \\ \left. \left. \frac{\bar{\rho}_v \delta^2}{\bar{\mu}_v} \hat{\phi}_T(\bar{h}, \zeta) \int_{\bar{\zeta}}^{\bar{h}} \bar{\bar{T}}_v(\zeta') d\zeta' \right] d\zeta \right\} \right] d\zeta \tag{21}$$

式中 $\Delta = \left[\frac{1}{\bar{\rho}_v J \delta^2} + \int_0^{\bar{h}} \hat{\phi}_T(\bar{h}; \bar{z}') d\bar{z}' \right]^{-1}$

$$\bar{\bar{T}}_v(\bar{z}) = \frac{\bar{T}_v(\bar{z})}{\bar{T}_0}。$$

真实桩的扭转角 $\phi(z)$ 与虚拟桩的扭转角

$\phi_v(z)$ 相等, 为

$$\phi(\bar{z}) = \frac{\bar{T}_0 \Delta}{\bar{\rho}_v J \delta^2} \left\{ \hat{\phi}_T(\bar{h}, 0) + \int_0^{\bar{h}} \left[\bar{T}_v(\bar{z}') \frac{\partial \hat{\phi}_T(\bar{h}, \bar{z}')}{\partial \bar{z}'} + \frac{\bar{\rho}_v \delta^2}{\bar{\mu}_v} \hat{\phi}_T(\bar{h}, \bar{z}') \right] \int_{\bar{z}'}^{\bar{h}} \bar{T}_v(\zeta) d\zeta d\bar{z}' \right\} - \frac{\bar{T}_0}{\bar{\mu}_v J} \int_{\bar{z}}^{\bar{h}} \bar{T}_v(\zeta) d\zeta \quad (22)$$

真实桩的扭矩 $\bar{T}(\bar{z})$ 为虚拟桩的扭矩与扩展半空间相应截面处的扭矩之和, 即

$$\begin{aligned} \bar{T}(\bar{z}) = & \bar{T}_v(\bar{z}) + [\bar{T}_0 - \bar{T}_v(\bar{h})] \hat{\tau}_{z\theta}(\bar{z}, 0) + \\ & \bar{T}_v(\bar{h}) \hat{\tau}_{z\theta}(\bar{z}, \bar{h}) - \int_0^{\bar{h}} \frac{\partial \bar{T}_v(\bar{z}')}{\partial \bar{z}'} \hat{\tau}_{z\theta}(\bar{z}, \bar{z}') d\bar{z}' - \\ & \bar{\rho}_v J \delta^2 \int_0^{\bar{h}} \phi_v(\bar{z}') \hat{\tau}_{z\theta}(\bar{z}, \bar{z}') d\bar{z}' \end{aligned} \quad (23)$$

式(21 ~ 23) 中的 $\hat{\phi}_T(\bar{z}, \bar{z}')$, $\hat{\gamma}_T(\bar{z}, \bar{z}')$, $\hat{\gamma}_1(\bar{z}, \bar{z}')$ 和 $\hat{\tau}_{z\theta}(\bar{z}, \bar{z}')$ 为影响函数(其表达式见附录 A)。

4 数值算例

由式(21) 求出 $\bar{T}_v(\bar{z})$ 后, 便可由式(22) 和(23) 求出单桩的扭矩和扭转角, 就可进一步分析其动力响应。为验证本文理论方法的正确性和计算精度, 取桩 - 土相对刚度 $\mu_p/\mu_1 = 500000$, 振动频率 $\delta = 0.00001$ 时的两层地基模型在不同桩长下的桩顶无量纲刚度系数 $K_T (= \frac{3}{16} \frac{\mu_0 \bar{T}_0}{\phi(0)})$, 并将该退

表 1 两种不同计算方法所得的桩顶刚度系数
Tab. 1 Comparison between present and Luco's results for torsional stiffness of rigid pile

h/a	$\mu_0 = 1$		$\mu_0 = 0.5$		$\mu_0 = 0.25$	
	本文解	Luco 解	本文解	Luco 解	本文解	Luco 解
2	6.10	6.14	3.58	3.64	2.23	2.35
3	8.42	8.53	4.83	4.84	2.91	2.95
4	10.83	10.90	6.00	6.03	3.52	3.55

化结果与 Luco 的刚性桩近似解析解答^[11] 进行比较, 其结果见表 1。

从表 1 可看出这两种计算结果吻合的很好, 这说明本文所提出的方法可用于分析成层地基中单桩的扭转振动问题。

为进一步说明地基土成层性对单桩动力响应的影响, 下面给出两层地基模型的数值解。对于两层以上地基模型, 可采用同样的方法进行分析。在计算中, 取桩 - 土参数为 $\rho_p/\rho_1 = 1.2$, $\mu_p/\mu_1 = 21.0$, 上层土的厚度为 h , 桩的长径比 $h/a = 10.0, 20.0$, 无量纲振动频率 $\delta = 2.0, 3.0$ 。为考察上、下土层相对强度对桩动力响应的影响, 计算中取 $\mu_0 = 0.6$ 和 1.2 。 $\mu_0 > 1.0$ 表示下层土强度比上层土好, 为好的持力层; 反之, $\mu_0 < 1.0$ 表示下层土强度比上层土低, 为差的持力层。计算结果如图 4 ~ 图 7 所示。

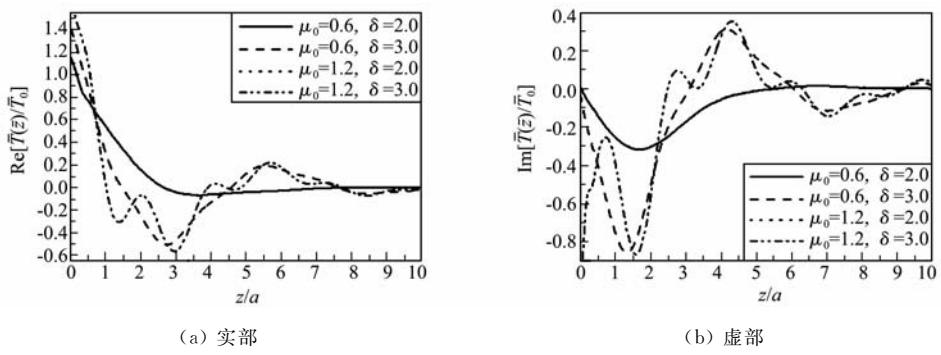


图 4 桩的扭矩随桩身的变化曲线 ($h/a = 10.0$)
Fig. 4 Curves of pile torque along pile body for $h/a = 10.0$

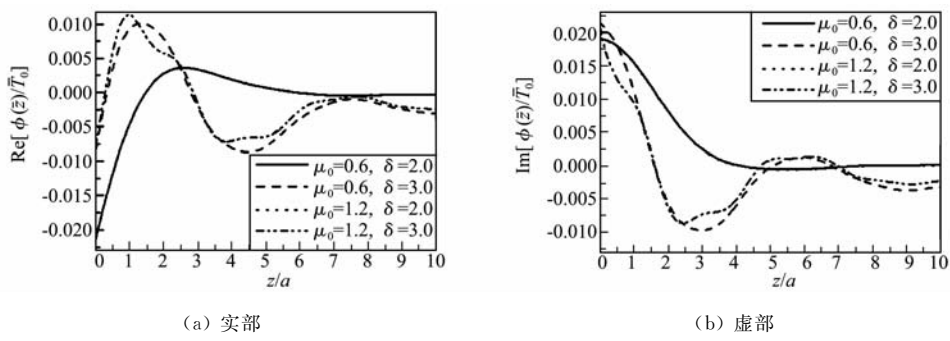


图 5 桩的扭转角随桩身的变化曲线 ($h/a = 10.0$)
Fig. 5 Curves of pile torsional angle along pile body for $h/a = 10.0$

从图 4 ~ 图 7 可看出:当振动频率较低时(如 $\delta \leq 2.0$),与均质地基类似^[12],桩的扭矩和扭转角随桩身的变化曲线是稳定的,没有出现振荡现象;而当振动频率较大时,如 $\delta \geq 3.0$,则出现较大的振荡现象。

对比图 4 ~ 图 7 可发现,在振动频率较低时(如 $\delta \leq 2.0$),桩端持力层好坏对桩的扭矩与扭转角没有多大影响,曲线基本上重合在一起,而当振动频率较大时,桩端持力层土的好坏对动力响应有一定的影响:持力层较差时,桩的扭矩和扭转角幅值是有规律地振荡,曲线比较光滑且沿着桩身逐渐衰减;而当持力层较好时,曲线振荡的非常剧烈,出现多个波峰与波谷相间情况。对比图 4 ~ 图 7 还可看出:桩的长径比对动力响应也有一定的影响:长

径比不同时,桩的扭矩和扭转角幅值曲线的波峰和波谷出现的位置不同,且其峰值也不同,长径比大时对应峰值较大,反之亦然。

在计算中,还计算了桩的无量纲动力柔度系数随剪切模量比的变化曲线。桩顶无量纲动力柔度系数 C_T 的定义为

$$C_T = \frac{16}{3} \phi(0) / \bar{T}_0$$

(24)

图 8 和图 9 是 $\mu_0 = 0.6$ 时桩顶的动力柔度系数随桩 - 土剪切模量比的变化曲线。从图中可看出:桩顶动力柔度系数的实部随剪切模量比的增大而逐渐减小,其虚部则相反,但曲线的变化规律是稳定的,不随振动频率的高低及桩的长径比的变化而出现振荡现象。从图 8 和图 9 可以看出,振动频

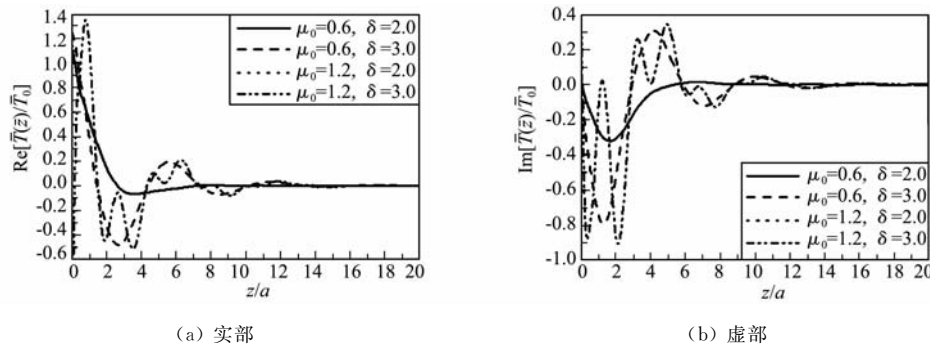


图 6 桩的扭矩随桩身的变化曲线($h/a = 20.0$)
Fig. 6 Curves of pile torque along pile body for $h/a = 20.0$

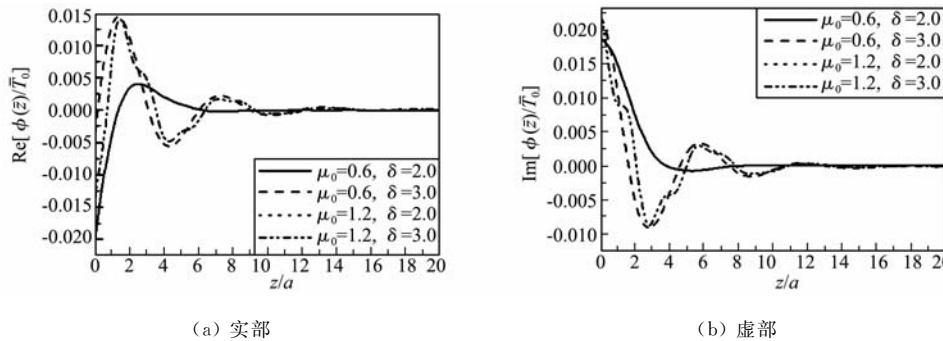


图 7 桩的扭转角随桩身的变化曲线($h/a = 20.0$)
Fig. 7 Curves of pile torsional angle along pile body for $h/a = 20.0$

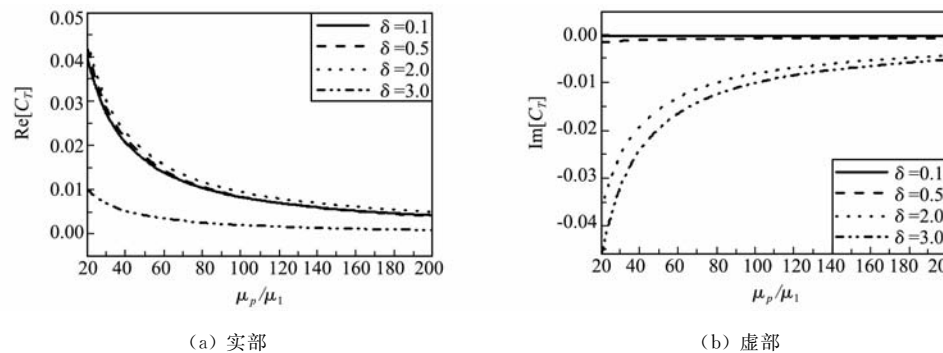
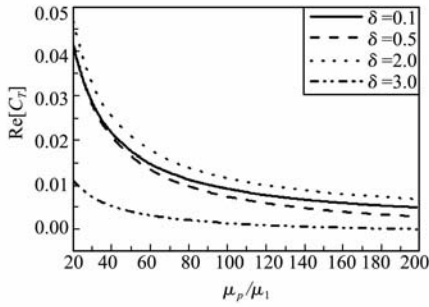
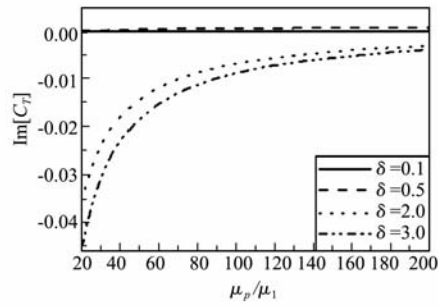


图 8 桩的柔度系数与桩 - 土剪切模量比的关系($h/a = 10.0$)
Fig. 8 Curves of pile compliance coefficient vs. pile-soil's shear modulus ratio for $h/a = 10.0$



(a) 实部



(b) 虚部

图 9 桩的柔度系数与桩-土剪切模量比的关系($h/a = 20.0$)Fig. 9 Curves of pile compliance coefficient vs. pile-soil's shear modulus ratio for $h/a = 20.0$

率与长径比对桩顶动力柔度系数有较大的影响。随着长径比的增大,动力柔度系数的实部和虚部逐渐减小,尤其在振动频率较高时,减小的幅度将更加明显。

5 结 语

本文从三维轴对称出发,采用 Muki 的方法并结合传递矩阵法,研究了层状地基中单桩的扭转振动问题,分析了桩-土参数和振动频率对动力响应的影响。研究结果表明:桩的扭矩、扭转角和动力柔度系数曲线在变化趋势上与匀质地基情况基本一致,但桩的长径比、桩土模量比和振动频率、地基土的层状性以及持力层土性质好坏对桩的动力响应有较大影响。所得结论可进一步加深桩-土共同作用和扭转波测桩机理的认识,对工程实践有一定的指导意义。

附录

影响函数为

$$\hat{\gamma}_T(\bar{z}, \bar{z}') = \frac{\partial \bar{u}_\theta(1, \bar{z}; \bar{z}')}{\partial \bar{z}} = \text{sgn}[\bar{z} - \bar{z}'] \hat{\gamma}_1(|\bar{z} - \bar{z}'|) + \hat{\gamma}_2(\bar{z}; \bar{z}') \quad (\text{A1})$$

$$\hat{\tau}_{\vartheta}(\bar{z}, \bar{z}') = 2\pi \int_0^1 \bar{r}^2 d\bar{r} \int_0^\infty p \frac{\partial \bar{u}_\theta^1}{\partial \bar{z}} J_1(p\bar{r}) dp \quad (\text{A2})$$

$$\phi_T^0(\bar{z}, \bar{z}') = \int_0^\infty p \bar{u}_\theta^1(1, \bar{z}; \bar{z}') J_1(p) dp \quad (\text{A3})$$

对于两层地基模型,上述影响函数可进一步化为

$$\begin{aligned} \hat{\gamma}_T(\bar{z}, \bar{z}') = & -\frac{1}{\pi(1+\mu_0)} \int_0^\infty \frac{J_1(p)J_2(p)}{H(p\bar{h})} \left[(1+\mu_0) \times \right. \\ & \left. \left(\frac{|\bar{z}' - \bar{z}|}{\bar{z}' - \bar{z}} e^{-j|\bar{z}' - \bar{z}|} - e^{-j(\bar{z}' + \bar{z})} \right) + \right. \\ & \left. (1 - \mu_0) (e^{-j(2\bar{h} - \bar{z}' - \bar{z})} - \right. \\ & \left. \left. \frac{|\bar{z}' - \bar{z}|}{\bar{z}' - \bar{z}} e^{-j(2\bar{h} - |\bar{z}' - \bar{z}|)} \right) \right] dp \quad (\text{A4}) \end{aligned}$$

$$\begin{aligned} \hat{\gamma}_1(\bar{z}, \bar{z}') = & \frac{1}{\pi(1+\mu_0)} \int_0^\infty \frac{J_1(p)J_2(p)}{H(p\bar{h})} \times \\ & [(1+\mu_0)e^{-j|\bar{z}' - \bar{z}|} - \\ & (1 - \mu_0)e^{-j(2\bar{h} - |\bar{z}' - \bar{z}|)}] dp \quad (\text{A5}) \end{aligned}$$

$$\begin{aligned} \hat{\tau}_{\vartheta}(\bar{z}, \bar{z}') = & -2 \int_0^\infty \frac{J_2^2(p)}{p(1+\mu_0)H(p\bar{h})} \left[(1+\mu_0) \times \right. \\ & \left. \left(\frac{|\bar{z}' - \bar{z}|}{\bar{z}' - \bar{z}} e^{-j|\bar{z}' - \bar{z}|} - e^{-j(\bar{z}' + \bar{z})} \right) + \right. \\ & \left. (1 - \mu_0) (e^{-j(2\bar{h} - \bar{z}' - \bar{z})} - \right. \\ & \left. \left. \frac{|\bar{z}' - \bar{z}|}{\bar{z}' - \bar{z}} e^{-j(2\bar{h} - |\bar{z}' - \bar{z}|)} \right) \right] dp \quad (\text{A6}) \end{aligned}$$

$$\begin{aligned} \hat{\phi}_T(\bar{z}, \bar{z}') = & -\frac{1}{\pi(1+\mu_0)} \int_0^\infty \frac{J_1(p)J_2(p)}{jH(p\bar{h})} \left[(1+\mu_0) \right. \\ & \left. (e^{-j|\bar{z}' - \bar{z}|} + e^{-j(\bar{z}' + \bar{z})}) + \right. \\ & \left. (1 - \mu_0) (e^{-j(2\bar{h} - |\bar{z}' - \bar{z}|)} + e^{-j(2\bar{h} - \bar{z}' - \bar{z})}) \right] dp \quad (\text{A7}) \end{aligned}$$

$$\text{式中 } H(p\bar{h}) = \frac{1 + \mu_0 - e^{-2j\bar{h}} + \mu_0 e^{-2j\bar{h}}}{1 + \mu_0}, \mu_0 = \frac{\mu_2}{\mu_1}.$$

参考文献 (References):

- [1] NOVAK M, NOGAMI T. Soil-pile interaction in horizontal vibration[J]. *Earthquake Engineering & Structural Dynamics*, 1977, **5**:263-281.
- [2] KUHLEMEYER RL. Vertical vibration of piles[J]. *Journal of Geotechnical Engineering*, 1979, **105**(2): 273-287.
- [3] SEN R, DAVIS TG, BANERJEE PK. Dynamic analysis of piles and pile groups embedded in homogeneous soil[J]. *Earthquake Engineering & Structural Dynamics*, 1985, **13**:53-65.
- [4] CHEUNG Y K, THAM LG. Transient response of single piles and pile groups[J]. *Journal of Geotechnical Engineering*, 1991, **117**(2):1860-1879.
- [5] FOWLER GF and SINCLAIR GB. The longitudinal harmonic excitation of a circular bar embedded in an elastic half-space[J]. *International Journal of Sol-*

ids and Structures, 1978, **14**:999-1012.

- [6] ZENG X and RAJAPAKSE RKND. Dynamic axial load transfer from elastic bar to poroelastic medium [J]. *Journal of Engineering Mechanics*, 1999, **125** (9):1048-1055.
- [7] 蒯行成, 沈蒲生. 层状土中单桩水平振动的动力阻抗[J]. 湖南大学学报, 1998, **25** (3): 51-54, 95. (KUAI Xing-cheng, SHEN Pu-sheng. Horizontal dynamic impedances of single piles in layered Soil[J]. *Journal of Hunan University*, 1998, **25** (3): 51-54, 55. (in Chinese))
- [8] 王宏志, 陈云敏, 陈仁朋. 多层土中桩的振动半解析解[J]. 振动工程学报, 2000, **13** (4):660-665. (WANG Hong-zhi, CHEN Yun-min, CHEN Ren-peng. Semi-analytical solution for response of pile in multi-layered soil under impact force and its simulation[J]. *Journal of Vibration Engineering*, 2000, **13** (4):660-665. (in Chinese))
- [9] 王立忠, 冯永正, 柯 瀚, 等. 瑞利波作用下成层地基中单桩横向振动分析[J]. 振动工程学报, 2001, **14** (2): 205-210. (WANG Li-zhong, FENG Yong-zheng, KE Han, et al. Lateral dynamic response of single pile in multiple layers soil during the passage of Rayleigh waves[J]. *Journal of Vibration Engineering*, 2001, **14** (2):205-210. (in Chinese))
- [10] 王 腾, 王奎华, 谢康和. 成层土中桩的纵向振动理论研究及应用[J]. 土木工程学报, 2002, **35** (1): 83-87. (WANG Teng, WANG Kui-hua, XIE Kang-he. Study on vibration properties of piles in layered soils [J]. *Chinese Journal of Civil Engineering*, 2002, **35** (1):83-87. (in Chinese))
- [11] LUCO J E. Torsion of a rigid cylinder embedded in an elastic half space[J]. *Journal of Applied Mechanics*, ASME, 1976, **43**:419-423.
- [12] 王国才. 单桩复合地基振动特性研究 [R]. 上海: 上海交通大学博士后出站报告, 2004. (WANG Guo-cai. Study on the vibration characteristics of single-pile composite foundation[R]. Postdoctoral research report of Shanghai Jiaotong University, 2004. (in Chinese))

Study on the torsional oscillations of single pile embedded in layered foundation

WANG Guo-cai*, DING Cui-hong, WANG Zhe, HUANG Jin

(School of Civil Engineering and Architecture, Zhejiang University of Technology, Hangzhou 310014, China)

Abstract: The torsional oscillations of single pile embedded in layered foundation are investigated by means of integral transformation and Muki's methods. In the analysis, the Hankel transformation is made on the governing equations firstly and the primary parameter solutions and stiffness matrix of single layered foundation are established. The recurrence matrix of layered foundation is obtained. Then, using the recurrence matrix, boundary condition and deformation compatibility conditions, the fundamental integral equation of the torsional oscillation of single pile embedded in layered foundations is established and programmed. At the end of the paper, the torsional responses of the rigid single pile in degenerated layered foundation are obtained and show good agreement with the existing theoretical solutions. Furthermore, the dynamic torsional responses of single pile in two layered foundation are analyzed by numerical examples and the influence of pile-soil parameters on the dynamic response is investigated. The conclusions obtained have some guidelines on practical engineering and torsional wave testing of pile foundation.

Key words: layered foundation; single pile; torsional oscillation; dynamic compliance; integral transformation